AVIRAL JEE | NEET | FOUNDATIONS

WEEKLY TEST OYJ MATHEMATICS SOLUTION 11 AUGUST 2019

31. (b) $\int (\sin^4 x - \cos^4 x)dx = \int (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)dx$

$= \int (\sin^2 x - \cos^2 x)dx = -\int (\cos^2 x - \sin^2 x)dx$

$= -\int \cos 2x\, dx = \frac{-\sin 2x}{2} + c$.

32. (c) $\int \frac{dx}{1-x^2} = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) + c = \frac{1}{2}\ \ln\left|\frac{1+x}{1-x}\right| + c.$

33. (c) Put $b\cos x = t$.

34. (a) Put $\log \sin x = t$.

35. (b) $\int \frac{x-2}{x(2\log x - x)}dx = -\int \frac{\left(\frac{2}{x}-1\right)}{(2\log x - x)}dx$

Now put $(2\log x - x) = t \Rightarrow \left(\frac{2}{x}-1\right)dx = dt,$ then it reduces to $-\int \frac{1}{t}dt = -\log t = -\log(2\log x - x)$

$= \log\left(\frac{1}{2\log x - x}\right) + c$.

36. (b) Put $\log x = t \Rightarrow \frac{1}{x}dx = dt,$ then

$\int \frac{1}{x(\log x)^2}dx = \int \frac{1}{t^2}dt = -\frac{1}{t} + c = -\frac{1}{\log x} + c.$

37. (b) $\int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

Now put $e^x + e^{-x} = t \Rightarrow (e^x - e^{-x})dx = dt,$ then it reduces to $\int \frac{dt}{t} = \log t = \log(e^x + e^{-x}) = \log(e^{2x}+1) - x + c$.

38. (b) $\int \cos^3 x\, e^{\log \sin x}dx = \int \cos^3 x \sin x\, dx$

$= -\int t^3 dt = -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$ {Putting $t = \cos x$}.

39. (b) $\int \sec^{2/3} x\, \text{cosec}^{4/3} x\, dx = \int \frac{dx}{\sin^{4/3} x \cos^{2/3} x}$

Multiplying N^r and D^r by $\cos^2 x,$ we get

{Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$}

$= \int \frac{\sec^2 x\, dx}{\tan^{4/3} x} = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + c = -3(\tan x)^{-1/3} + c$.

40. (c) $\int \frac{\sin 2x}{\sin 5x \sin 3x}dx = \int \frac{\sin(5x-3x)}{\sin 5x \sin 3x}dx$

$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x}dx$

$= \frac{1}{3}\log \sin 3x - \frac{1}{5}\log \sin 5x + c.$

41. (b) $\int \sin^3 x . \cos x \, dx$. Put $\sin x = t$, then

$\cos x \, dx = dt$; $\int t^3 dt = \frac{t^4}{4} = \frac{\sin^4 x}{4} + c$.

42. (c) Put $x^2 + \sin 2x + 2x = t$, then it reduces to

$\frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c = \frac{1}{2}\log(x^2 + \sin 2x + 2x) + c.$

43. (a) Put $x = \sin\theta \Rightarrow dx = \cos\theta \, d\theta$, then it reduces to $\int (1 + \sin^2\theta) d\theta = \theta + \frac{1}{2}\int (1 - \cos 2\theta) d\theta$

$= \frac{3\theta}{2} - \frac{1}{2}\sin\theta\sqrt{1-\sin^2\theta} + c = \frac{3}{2}\sin^{-1} x - \frac{1}{2}x\sqrt{1-x^2} + c$.

44. (a) $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$

Now put $\sin x + \cos x = t$, then the required integral is

$-\frac{1}{\sin x + \cos x} + c$.

45. (c) Put $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$, then

$\int \frac{dx}{(x^2-1)\sqrt{x^2+1}} = \int \frac{\sec^2\theta \, d\theta}{(\tan^2\theta - 1)\sec\theta} = \int \frac{\cos\theta \, d\theta}{(2\sin^2\theta - 1)}$

Again put $t = \sin\theta \Rightarrow dt = \cos\theta \, d\theta$, then it reduces to

$$\int \frac{dt}{(2t^2-1)} = \frac{1}{2}\int \frac{dt}{t^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{2\sqrt{2}}\log\left(\frac{t - \frac{1}{\sqrt{2}}}{t + \frac{1}{\sqrt{2}}}\right) + c$$

$$= \frac{1}{2\sqrt{2}}\log\left(\frac{\sqrt{1+x^2} - x\sqrt{2}}{\sqrt{1+x^2} + x\sqrt{2}}\right) + c.$$